F. William Lawvere - *Are homotopy types the same as infinitesimal skeleta?*

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Among the functors $\mathcal{S} \to \mathcal{X}$ between cartesian-closed categories which have both left and right adjoints, there are those special ones in which those two adjoints are isomorphic, *i.e.* in which so to speak every component of an object *X* contains a unique point of *X*. For example, $\mathcal{S}$ could be the classifying topos for algebraic field extensions of a given field while $\mathcal{X}$ is the classifying topos for strongly local algebras over the same base; or $\mathcal{X}$ could be the category of presheaves for a category in $\mathcal{S}$ which has a terminal object which is also initial. Such could obviously arise by taking a suitable part $\mathcal{X} = \mathrm{Infl}(\mathcal{Z}) \to \mathcal{Z}$ of a large $\mathcal{Z}$ on which the two adjoints do not necessarily agree. But an opposite way to get from $\mathcal{Z}$ to such a special $\mathcal{X}$ arises by the Hurewicz construction for the case where the left adjoint from $\mathcal{Z}$ to $\mathcal{S}$ preserves finite products: the resulting re-enrichment $\mathcal{Z} \to \mathcal{H}_o(\mathcal{Z}) = \mathcal{X}$ will satisfy our special equation if the left adjoint actually preserves infinite ($\mathcal{S}$-indexed) products, which is to be expected if components can be detected using continuous intervals but not if only combinatorial intervals are available in $\mathcal{Z}$. Thus, although considerable fine-tuning of the setting (over fixed $\mathcal{S}$) is still needed, it appears that the answer to the question in the title is No, they are identical opposites.