als die in allen solchen Kalkülen als gültig ausgezeichneten Schlüsse definiert und es wird gezeigt, daß dadurch genau die gültigen Schlüsse der intuitionistischen Aussagenlogik erfaßt werden. (*Received June 28, 1966.*)

HANS LÄUCHLI. *A decision procedure for the weak second order theory of linear order.* The weak second order (WS) language of a relational system contains besides the first order symbols special variables which range over finite subsets of the field of the system, and a predicate $\in$ for membership. R. Büchi proved the decidability of the WS theory of the class of well-orderings. We establish the corresponding result for the class of arbitrary linearly ordered sets. The WS theory is viewed as a first order theory with a restriction with regard to the models. The Fraïssé method can be applied in a similar way as in (*) "On the elementary theory of linear order" by Läuchli and Leonard. The crucial part is based on an observation by F. Galvin, namely, that the results of (*) can be established without using the completeness theorem.

If WS is understood in the sense of A. Tarski—finite sequences instead of sets—then the corresponding theory of linear order is easily seen to be undecidable. As to general second order theory—arbitrary subsets—the problem seems to be open. Our proof does not generalize. (*Received August 8, 1966.*)

F. W. LAWVERE. *Theories as categories and the completeness theorem.* Simplifying the definition given at the Leicester Conference (this JOURNAL, 1966), an *elementary theory* is any small category T equipped with finite categorical products satisfying the following three conditions: (1) There is a distinguished object B with two distinguished morphisms $1 \to B$ called "true" and "false" (1 being the empty product) such that for all objects X, $\langle \text{true}, X\rangle$ and $\langle \text{false}, X\rangle$ have the universal mapping property of injections for a categorical coproduct—briefly, $B \times X \cong X + X$, so that in particular $B \cong 1 + 1$; this first condition has the consequence that the set $T(X, B)$ of morphisms $X \to B$ has functorially the structure of a Boolean algebra, whose order relation will be called $\vdash_X$. (2) For any $f\colon X \to Y$, $\varphi\colon X \to B$ in T, there is $\exists_f[\varphi]\colon Y \to B$ in T such that for all $\psi\colon Y \to B$, $\exists_f[\varphi] \vdash_Y \psi$ iff $\varphi \vdash_X f\psi$; in case f is a projection, this property yields the usual notion of existential quantification, and in case f is the diagonal $X \to X \times X$ while $\varphi = \text{true}_X$, this condition yields $\mathit{equality}_X\colon X \times X \to B$. (3) There is a further distinguished object A such that the categorical product induces a one-one correspondence of the words in A, B onto the objects of T; morphisms $A^n \to A$ are then thought of as n-ary *terms* and morphisms $A^n \to B$ are thought of as n-ary *formulas* so that in particular $T(1, B)$ is the set of *sentences* of T. A *model* of an elementary theory T is any product preserving functor $M\colon T \to \text{Sets}$ which takes B to a two-element set and which takes each $\exists_f[\varphi]$ to the indicator of the image under fM of the subset whose indicator is φM. By the usual reduction to the Consistency Theorem in the initial step and an application of the Prime Ideal Theorem in the final step, the proof of the *Completeness Theorem* is reduced to the following three facts: (α) By any *morphism* of *theories* $T \to T'$ (obvious definition) any model of T' yields a model of T. (β) The *representable functor* $T'(1, -)\colon T' \to \text{Sets}$ is a model for T' iff (β_1) $T'(1, B) = \{\text{true}, \text{false}\}$ and (β_2) given $f\colon X \to Y$ and $\varphi\colon X \to B$ in T', then for all $y\colon 1 \to Y$ in T', $y\exists_f[\varphi] = \text{true}_Y$ iff there exists $x\colon 1 \to X$ in T' such that $xf = y$ and $x\varphi = \text{true}_X$. (γ) By the Adjoint Functor Theorem of general categorical algebra $T \to T'$ where T' has property (β_2) can be constructed as the adjunction of the adjoint to a "forgetful" functor to the category of elementary theories from that of *τ-Theories* defined below. Replace (2) by the requirements that equality exist and that for $f\colon X \to Y$, $\varphi\colon X \to B$ there is given $\tau_f[\varphi] = g\colon Y \to X$ such that $fgf = f$ and $\varphi \vdash_X fg\varphi$; the forgetful functor is defined by $\exists_f[\varphi] = g\varphi \wedge \langle gf, Y\rangle eq_Y$. Verification that this $\exists$ satisfies (2) and that every τ-theory satisfies (β_2) is straightforward. (*Received August 12, 1966.*)

HILBERT LEVITZ. *On Finsler's hierarchy of generalized arithmetical operations.* Finsler set up a hierarchy $\{f_a\}$ of binary operations defined on the ordinals; ***Commentarii mathematici Helvetici***, vol. 25 (1951), p. 75. In particular, $f_0(b, c) = c + 1$, $f_1(b, c) = c + b$, $f_2(b, c) = c \cdot b$, and $f_3(b, c) = c^b$. The hierarchy has the property that $f_{a+1}(b + 1, c) = f_a(c, f_{a+1}(b, c))$; this generalizes the recursion formulae: $c + (b + 1) = (c + b) + 1$, $c \cdot (b + 1) = c \cdot b + c$, $c^{b+1} = c^b \cdot c$.

He then described a certain number κ with the property that if $\omega < \alpha < \kappa$, then α can be