If $p \varepsilon A$ is of support one, let $fr(p, M)$, $M \varepsilon S_0$, denote the relative frequency of (the property) p in M.

Theorem. $\frac{1}{n} \Sigma\, fr(p, M_i)$ *converges almost certainly to* $c(p)$. The proof uses the following facts: (i) every c on A can be represented in the form $c(p) = \int \mu_M(p[M])\, d\lambda(M)$, where λ is a σ-additive probability measure on S, μ_M a probability on the sets $p[M] = \{x \varepsilon X_M^I : p_M(x) = 1\}$; (ii) $\mu_M(p[M]) = fr(p, M)$ if $M \varepsilon S_0$; and (iii) $\lambda(S_0) = 1$. The proof then consists in an application of the strong law of large numbers. (*Received 23 July, 1965.*)

R. B. JENSEN. *Independence of the axiom of dependent choices from the countable axiom of choice.*

If ZF is consistent, then it remains consistent after adjoining the following axioms:

(1) The countable axiom of choice,

(2) There is a relation R such that for every real x a real y exists such that: Rxy but there is no sequence $\{x_i : i < \omega\}$ such that: Rx_ix_{i+1} for all i. (*Received 11 September, 1965.*)

CAROL KARP. *Primitive recursive set functions: a formulation with applications to infinitary formal systems.*

Call a set function $F(x_1, \ldots, x_n)$ finitarily primitive recursive (f.p.r.) if it arises by composition, set recursion, and the set formation schema $F(x_1, \ldots, x_n) = \{G(z, x_1, \ldots, x_n) \mid z \varepsilon x_1\}$, from a set of initial functions consisting of the projections, the pairing function, union, difference, ω and 0. Gödel's F-function generating the constructible sets is an example of an f.p.r.function (f.p.r.f.).

Let Ω be a regular, non-denumerable cardinal and let T_Ω be the set of all sets hereditarily of power less than Ω. Then T_Ω is closed under f.p.r.f.'s and the Löwenheim-Skolem Theorem for (Ω, ω)-languages implies that for $x_1, \ldots, x_n \varepsilon T_\Omega$, if R is a f.p.r. relation and $(\exists y)R(x_1, \ldots, x_n, y)$ holds in V then it holds in T_Ω. Applying this to Gödel's F, we have yet another proof that $V = L$ implies the generalized continuum hypothesis. The Ω-primitive recursive functions (Ω-p.r.f.'s) are restrictions to T_Ω of functions $F(x_1, \ldots, x_n) = G(x_1, \ldots, x_n, s_1, \ldots, s_m)$, $s_1, \ldots, s_m \varepsilon T_\Omega$, G an f.p.r.f. Proof predicates for the basic Ω-formal systems as well as computation predicates for the Ω-recursive functions of ordinals, are Ω-p.r. The Ω-p.r.f.'s can be represented in a simple (Ω, ω)-formal system with an Ω-p.r. proof predicate. The enumeration theorem follows for Ω-recursively enumerable (Σ_1-Ω-p.r.) predicates.

For infinite cardinals κ, the κ^+-propositional formulas are like ordinary propositional formulas except that conjunctions and disjunctions of length $< \kappa^+$ are admitted. *Theorem* 1. If κ is a strong limit cardinal of cofinality ω, then the κ^+-tautologies are κ^+-p.r. axiomatizable. *Theorem* 2. If $\kappa \exp \omega = \kappa$, then the κ^+-tautologies are not κ^+-p.r. axiomatizable. Thus assuming the generalized continuum hypothesis, the κ^+-tautologies are κ^+-p.r. axiomatizable iff $cf(\kappa) = \omega$. The methods are sufficiently general that these theorems could be expected to hold for any reasonable notion of κ^+-recursiveness. (*Received 20 July, 1965.*)

F. WILLIAM LAWVERE. *Functorial semantics of elementary theories.*

By an elementary theory we mean a small category T with the following structure: a) There are two distinguished objects A, L and all objects are precisely the finite categorical products $A^n \times L^k$ b) For each object X, $L^k \times X$ is the 2^k-fold coproduct (sum) of X with itself; e.g. $L = 1 + 1$. c) The 2^k-fold product of X with itself is the categorical exponential "X to the power L^k" [The more precise statement of a), b), c) is that there are given projections, injections, and evaluation maps satisfying usual universal mapping properties.] d) For every pair $f : X \to Y$, $\varphi : X \to L$ in T there is $\exists_f[\varphi] : Y \to L$ such that for every $\psi : Y \to L$ one has $\exists_f[\varphi] \leq_Y \psi$ iff $\varphi \leq_X f\psi$. Here $\leq_X$ is the partial order of a canonical Boolean structure which the set of all maps $X \to L$ enjoys by virtue of a), b), c). Every "applied first-order 'theory' with equality" gives rise to an elementary theory in which (equivalence classes of) n-ary terms become maps $A^n \to A$ and n-ary formulas become maps $A^n \to L$; the above conditions imply that all maps in T are in one way or another "tuples" of maps of these two special kinds. Other consequences include the existence of a map $A^2 \to L$ playing the role of equality, and for every sentence $\varphi : 1 \to L$ a corresponding binary function symbol $A^2 \to A$ which in any model denotes one or other of the projections depending on the truth or falsity of φ in the model. Every instance of $\exists_f[\varphi]$ is equal to a certain logical combination of special instances which correspond to the usual quantifiers (namely instances in which f is a projection $A^{n+1} \to A^n$). A model is a functor $M : T \to \mathscr{S}$ to the category $\mathscr{S}$ of sets (Lawvere ***PNAS*** Dec. 1964) which preserves the additional structure (in particular (L)M is a two element set and $(\exists_f[\varphi])M$ is the indicator of the image under (f)M of the subset whose indicator is $(\varphi)M$; the set (A)M is the universe of the model. Natural transformations between models are exactly elementary embeddings, yielding an "elementary category" $\mathscr{S}^{[T]}$. Any morphism $T' \to T$ of theories yields an "elementary functor" $\mathscr{S}^{[T]} \to \mathscr{S}^{[T']}$ which preserves universes ("reducts" are special cases of elementary functors). Interpreting implicit definability in terms of natural transformations yields a functor "elementary structure" adjoint to the single large functor "elementary semantics" defined by the foregoing sentence. Given a first-order language-with-axioms $\mathscr{L}$, the associated theory T may be thought of as the "*Sinn*" of $\mathscr{L}$ and the category $\mathscr{S}^{[T]}$ as the "*Bedeutung*" of $\mathscr{L}$; a similar remark applies to the algebraic theories previously discussed (Lawvere, ***PNAS***, Nov. 1963) and to a notion of "hypertheory" now being investigated. From this approach results a canonical procedure for embedding elementary categories and hypercategories as *full* subcategories of equationally definable categories of algebras. (*Received 17 Aug., 1965.*)

THEODORE A. LINDEN. *Equivalences between Gödel's definitions of constructibility.* The definitions of constructibility given by Kurt Gödel in his paper on the consistency of the continuum hypothesis and in his later monograph are different. But they can be proved equivalent not only for the class of all constructible sets, but also at each ω_γ, where ω_γ is the least ordinal of cardinality γ. That is, we always have $M_{\omega_\gamma} = F(\omega_\gamma)$. The proof is by transfinite induction on the statement, "For $\alpha < \omega_\gamma$ there exists an $\alpha' < \omega_\gamma$ such that $M_\alpha = F(\alpha')$." Take $\alpha_1 < \omega_\gamma$, we show that for any set A which is first order definable over $F(\alpha_1)$, there is an $\alpha_2 < \omega_\gamma$ such that $A = F(\alpha_2)$. It then follows that there is an $\alpha_3 < \omega_\gamma$ such that $\mathfrak{F}\ (F(\alpha_1))$, the class of everything first order definable over $F(\alpha_1)$, is a subset of $F(\alpha_3)$. Furthermore α_3 can be chosen so that $\mathfrak{F}\ (F(\alpha_1))$ is actually first order definable over $F(\alpha_3)$, and hence there is an $\alpha_4 < \omega_\gamma$ such that $\mathfrak{F}\ (F(\alpha_1))$ equals $F(\alpha_4)$. This gives the induction from α to $\alpha + 1$. Similar arguments will take care of the induction at limit ordinals. $M_{\omega_\gamma} \subseteq F(\omega_\gamma)$ then follows by induction and the nature of the function F. The converse is easy. (*Received 15 July, 1965.*)

JOAN RAND MOSCHOVAKIS. *Disjunction and existence in formalized intuitionistic analysis.*

For the common portion of intuitionistic and classical analysis, as formalized by Kleene in S. C. Kleene and R. E. Vesley, ***Foundations of intuitionistic mathematics,*** Amsterdam (North Holland) 1965, we obtain the following.

Theorem. For closed formulae A, B, $\exists xA(x)$: